Grading Scheme

IGO 2021

8th Iranian Geometry Olympiad

Elementary Level

Problem 1.

(8 points) Drawing the desired figure.

- **(4 points)** Drawing a figure with 1 reflection Symmetry.

Problem 2.

(2 points) Considering the point N' on AD such that $LN' \parallel AB$.

(2 points) Proving that $\left[KLMN'\right] = 1/2 \left[ABCD\right]$.

(4 points) Proving that $KM \parallel AD$.

Problem 3.

(2 points) Proving the lemma 1.

(6 points) Investigate the case 2.

- **(2 points)** Showing that $\angle P'ZQ' = 360^\circ - 2\angle P'BQ'$.
- **(2 points)** Showing that $\angle PXQ = 180^\circ - \angle PBQ$.
- **(2 points)** Calculating the angle $\angle PBQ$.

(0 point) Investigate the case 1.

Problem 4.

- **Solution 1:**

(2 points) Considering the point Z and showing that $ZB = DX = FY$.

(2 points) Calculating the angle $\angle OZB$.

(2 points) Calculating the angles $\angle ODX$ and $\angle OFY$.

(2 points) Proving that $OZB \cong ODX \cong OFY$.

- **Solution 2:**

  (2 points) Considering the points K, L, T.

  (2 points) Proving that $ABE \cong AXK$.

  (2 points) Proving that $ABD \cong ALY$.

  (2 points) Proving the equality of power of points B, X, Y and finishing the solution.

Problem 5.

(3 points) Conclude the first equality.

(2 points) Conclude the second equality.

(3 points) Finishing the solution.

Intermediate Level

Problem 1.

(3 points) Considering the centroid (G) and showing that $GD \parallel EC$.

(1 point) Proving that $HB \perp GD$.
(1 point) Proving that $GH \perp BC$.

(3 points) Proving that H is orthocenter of triangle BGD and finishing the solution.

Problem 2.

- **Solution 1:**

(1 point) Proving that $DAF \sim BEC$.

(2 points) Proving that $EF \parallel AD \parallel BC$.

(2 points) Considering the point F' and showing that $AF'BF$ is cyclic.

(1 point) Proving that $AE \cdot EB = BC^2$.

(2 points) Proving the inequality.

- **Solution 2:**

(1 point) Considering the points X and Y and showing that $DYBF$ and $CXAF$ are cyclic.

(2 points) Calculating the CD^2 (equation 1).

(1 point) Calculating the DX and CY.

(2 points) Proving the inequality $DX + CY \geq 4BC$.

(2 points) Finishing the solution.

Problem 3.

(2 points) Considering the point K' such that $ABC \sim AK'D$.

(1 point) Showing that $DC = DE$.

(2 points) Proving that $ABK'F$ is cyclic.

(1 point) Proving that $ELDM$ is cyclic.

(2 points) Showing that the points K, L, M are collinear.

Problem 4.

(1 point) Considering the point Q'.

(1 point) Showing that the point M lies on perpendicular bisector of CI.

(1 point) Proving that $Q'I \perp MI$.

(2 points) Proving that $MIQ' \sim MNC$.

(1 point) Proving that $ST \parallel IN$.

(2 points) Showing that the points T, C, Q' are collinear.

Problem 5.

(5 points) Proving that the circles XLD and CXK pass through a fixed point.

- **(3 points)** Considering the fixed points L' and D'.
- **(2 points)** Proving that $D'XLD$ is cyclic.

(3 points) Showing that the line XY passes through a fixed point.

- **(1 point)** Parallel case.
- **(2 points)** Concurrent case.

Advanced Level

Problem 1.

(3 points) Showing that $\angle EFA = \angle B - \angle C$.

(3 points) Proving that $AEHF$ is cyclic.

(2 points) Finishing the solution.

Problem 2.

(1 point) Redefine the point F.

(2 points) Proving that CP is tangent to the circumcircle of triangle CBD.

(2 points) Calculating the angle of $\angle FBC$.

(3 points) Showing that $\angle ABC = 2\angle AFC$ and Conclusion.

Problem 3.

(2 points) Just to prove that the two circles are tangent (with any solution).

(1 point) Pointing to the idea of Inversion centered at H.

(2 points) Constructing the points P', T', L'.

(2 points) Showing that the $P'K$ is the image of ω under the inversion and $P'K \perp BC$.

(1 point) Proving that MN is the perpendicular bisector of DL'.

(1 point) Proving that $DL'KT'$ is cyclic by center M.

(1 point) Finishing the solution.

Problem 4.

(3 points) First Lemma.

(3 points) Second Lemma.

(2 points) Finishing the solution.

Problem 5.

(2 points) Proving that the point A lies on Euler line.

- **(1 point)** Proving that the points H, O, A' are collinear.
- **(1 point)** Proving that the points A, O, A' are collinear.

(6 points) Proving that the point D lies on Euler line.

- **(2 points)** Proving that $A'IKE$ is cyclic.
- **(2 points)** Proving that IXY is isosceles.
- **(1 point)** Proving that $OXY \sim HFE$.
- **(1 point)** Proving that the points H, O, D are collinear.